ALFVÉN-CYCLOTRON FLUCTUATIONS: LINEAR VLASOV THEORY

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23 March 2004

Abstract

Linear Vlasov dispersion theory for a homogeneous, collisionless electron-proton plasma with Maxwellian velocity distributions is used to examine the damping of Alfvén-cyclotron fluctuations. Fluctuations of sufficiently long wavelength are essentially undamped, but as $k_{\parallel}$, the wavevector component parallel to the background magnetic field $\mathbf{B}_o$, reaches a characteristic dissipation value k_d, the protons become cyclotron resonant and damping begins abruptly. For proton cyclotron damping, $k_d c/\omega_p \sim 1$ for $10^{-3} \lesssim \beta_p \lesssim 10^{-1}$ where $\beta_p \equiv 8\pi n_p k_B T_p/B_o^2$ and ω_p/c is the proton inertial length. At $k_{\parallel} < k_d$, $m_e/m_p < \beta_e$ and $\beta_p \lesssim 0.10$, the electron Landau resonance becomes the primary contributor to fluctuation dissipation, yielding a damping rate which scales as $\omega_r\sqrt{\beta_e}(k_{\perp}c/\omega_p)^2$ where ω_r is the real frequency and $k_{\perp}$ is the wavevector component perpendicular to $\mathbf{B}_o$. As β_p increases from 0.10 to 10, the proton Landau resonance makes an increasing contribution to damping of these waves at $k_{\parallel} < k_d$ and $0^o < \theta < 30^o$ where $\theta = \arccos(\hat{\mathbf{k}} \cdot \hat{\mathbf{B}}_o)$. The maximum damping rate due to the proton Landau resonance scales approximately as $\beta_p(kc/\omega_p)^2$ over $0.50 \leq \beta_p \leq 10$. Both magnetic transit-time damping and electric Landau damping may contribute to Landau resonant dissipation; in the electron Landau resonance regime the former is important only at propagation almost parallel to $\mathbf{B}_o$, whereas proton transit-time damping can be relatively important at both quasi-parallel and quasi-perpendicular propagation of Alfvén-cyclotron fluctuations.

1. Introduction

Understanding the dissipation of turbulent magnetic fluctuations in the collisionless plasmas of space and astrophysics is fundamental to comprehending not only the properties of the turbulence itself, but also the processes by which plasma species are heated. In the turbulence scenario, large-amplitude, long-wavelength magnetic fluctuations undergo nonlinear processes which cascade their energy to successively shorter wavelengths, leading to an ensemble of fluctuations with random phases and a broad range of wavevectors; that is, magnetic turbulence. MHD simulations in homogeneous, collisionless, magnetized plasmas [*Biskamp and Müller*, 2000] as well as observations in the much less ideal solar wind [e.g., *Matthaeus and Goldstein*, 1982] support the picture that, at relatively long wavelengths, magnetic turbulence satisfies the classic Kolmogorov picture of fluid turbulence so that the magnetic power spectrum is approximately proportional to $k^{-5/3}$; this wavenumber regime is usually termed the "inertial range." In the Kolmogorov picture the turbulent energy cascade rate γ_c increases with wavenumber as a relatively weak function of k; for example [See Appendix]:

$$\gamma_c \sim a_c k^{2/3} \tag{1}$$

Here a_c is a dimensionless parameter that is a function of the turbulence amplitude at long wavelengths.

Turbulent magnetic fluctuations are subject not only to the cascade process, but also to collisionless damping which transfers their energy to the plasma particles. In the inertial range, the fluctuation damping rate γ is generally much smaller than the energy cascade rate. However, $|\gamma(k)|$ increases more steeply with k than γ_c, so that the two quantities become equal at some sufficiently large k that depends on the integrated amplitude of the turbulence. At still larger k, $|\gamma| > \gamma_c$, damping overwhelms the cascade, the magnetic power spectra decrease more rapidly than $k^{-5/3}$, and the fluctuations are said to be in the "dissipation range." Although magnetic power spectral properties in the inertial range seem to be relatively robust and independent of the cascade processes, the properties of dissipation range spectra almost certainly depend upon the details of the damping. The turbulent cascade usually leads to $|\delta B|/B_o << 1$ (Here $\mathbf{B}_o$ is the background magnetic field) in the dissipation range so that, in this regime, the fluctuations may be approximated as normal modes of the plasma, and linear theory is appropriate for describing their dispersion properties.

At wavelengths of the order of or longer than the thermal proton gyroradius, at least three normal modes can propagate in a homogeneous, isotropic, collisionless plasma: the Alfvén-cyclotron, the magnetosonic-whistler, and the ion acoustic (also called the "slow") modes [*Gary*, 1993]. The latter is heavily damped unless $T_e >> T_p$; as this condition does not hold in many space plasmas of interest, we do not study this mode here. Although the other two modes both contribute to observed solar wind magnetic fluctuation spectra, we focus here on the Alfvén-cyclotron fluctuations.

The literature on the linear kinetic theory of Alfvén-cyclotron modes in collisionless electron-proton plasmas includes *Barnes* [1966], *Gary* [1986], *Stix* [1992], *Krauss-Varban et al.* [1994], *Lysak and Lotko* [1996], *Leamon et al.* [1999], and *Cranmer and van Ballegooijen* [2003]. Such theories have been the basis of a number of quasilinear models for Alfvén-cyclotron wave heating and acceleration of the solar corona and solar wind; for examples see the extensive citation lists of *Hollweg and Isenberg* [2002], or *Gary and Saito* [2003]. Computer simulations which have addressed the interactions between an imposed spectrum of Alfvén-cyclotron fluctuations and the protons or electrons of a collisionless plasma include *Tanaka et al.* [1987, 1989], *Geary et al.* [1990], *Liewer et al.* [2001], *Ofman et al.* [2002], *Gary and Saito* [2003], and *Gary and Nishimura* [2004]. From such research, it is well established that, at sufficiently large wavevector components parallel to $\mathbf{B}_o$, the primary damping mechanism of these modes is the proton cyclotron resonance, but that, as the wavevector $\mathbf{k}$ is made more oblique to the background magnetic field, this interaction becomes nonresonant and the Landau resonance becomes the primary means of wave-particle dissipation.

A series of papers by *Leamon et al.* [1998a, 1998b, 1999, 2000] analyzed solar wind observations of magnetic fluctuations and drew conclusions about the dissipation range of magnetic turbulence in that medium. In order to provide a more complete theoretical understanding of dissipation range physics, we here have used linear Vlasov theory to quantify the cyclotron and Landau resonance damping of Alfvén-cyclotron fluctuations, and to develop scaling laws for $\gamma(\mathbf{k})$ which may be used to represent the consequences of such damping in turbulent cascade models. We consider a homogeneous, isotropic, collisionless plasma. In such a medium the Vlasov equation is generally regarded as providing an appropriate description for plasma dynamics. We here solve the full linear Vlasov dispersion equation using the formalism described in Chapters 5 and 6 of *Gary* [1993], and no approximations are made with respect to either plasma or fluctuation parameters.

We consider only Alfvén-cyclotron fluctuations, which have the approximate dispersion equation $\omega_r \simeq k_\parallel v_A$ and an upper bound to the real frequency of $\omega_r < \Omega_p$. At $\mathbf{k} \times \mathbf{B}_o = 0$ this mode is left-hand circularly polarized, is very weakly damped at long wavelengths, and becomes subject to strong proton cyclotron damping at a characteristic dissipation wavenumber k_d [e.g., *Gary*, 1999]. At $\mathbf{k} \times \mathbf{B}_o \neq 0$ the Alfvén-cyclotron mode is characterized by relatively small compressibility at all directions of propagation, but the polarization can change to right-hand elliptical at sufficiently oblique propagation [*Gary*, 1986]. At $k_\parallel < k_d$ and $m_e/m_p \lesssim \beta_e$ the Landau wave-electron interaction becomes an important source of damping at sufficiently oblique propagation. In this regime, Alfvén-cyclotron fluctuations are known as "kinetic Alfvén waves." Some representative linear dispersion plots of ω as a function of $k_\parallel c/\omega_p$ are illustrated in *Gary and Nishimura* [2004].

Throughout this manuscript we consider an electron-proton plasma and denote electrons with the subscript e and protons by p. The subscripts $\parallel$ and $\perp$ denote directions parallel and perpendicular to the background magnetic field. For the jth species we define $\beta_j \equiv 8\pi n_j k_B T_j / B_o^2$; the plasma frequency, $\omega_j \equiv \sqrt{4\pi n_j e_j^2/m_j}$; the cyclotron frequency, $\Omega_j \equiv e_j B_o / m_j c$; and the thermal speed, $v_j \equiv \sqrt{k_B T_{\parallel j}/m_j}$. The Alfvén speed is $v_A \equiv B_o/\sqrt{4\pi n_p m_p}$. The complex frequency is $\omega = \omega_r + i\gamma$, the Landau resonance factor of the jth species is $\zeta_j \equiv \omega/\sqrt{2}|k_\parallel|v_j$, and the cyclotron resonance factors of the jth species are $\zeta_j^\pm \equiv (\omega \pm \Omega_j)/\sqrt{2}|k_\parallel|v_j$. We define θ as the angle between $\mathbf{k}$ and $\mathbf{B}_o$, so that $\hat{\mathbf{k}} \cdot \hat{\mathbf{B}}_o = \cos(\theta)$.

2. Cyclotron resonance and Landau resonance regimes

This section describes our use of linear Vlasov theory to determine three distinct parameter regimes for which three distinct wave-particle resonances become the most important damping mechanisms for Alfvén-cyclotron fluctuations. Isotropic Maxwellian velocity distributions are assumed for both species. Unless stated otherwise, here and in Section 3 we use the following dimensionless parameters: $m_p/m_e = 1836$, $v_A/c = 10^{-4}$, and $T_e = T_p$. The last condition implies $\beta_p = \beta_e$ and we use these two symbols interchangeably unless we explicitly state that we have considered possible T_e/T_p variations.

Figure 1 presents contour plots of the damping rate γ/Ω_p as a function of the perpendicular and parallel components of the wavevector for five different values of β_p. First, for all five cases, for sufficiently large $k_\parallel c/\omega_p$ the damping rate becomes a weak function of $k_\perp$; that is the contours of constant γ/Ω_p are approximately vertical. Below we argue that this is the regime in which proton cyclotron damping is the dominant dissipation mechanism; we call this the "proton cyclotron regime." Second, at $\beta_p = 0.001$ and 0.01, there is a regime of $k_\parallel c/\omega_p$ for which the contours are approximately horizontal; that is, the damping rate is approximately a function of $k_\perp$ alone. Here damping at the electron Landau resonance dominates, and we call this the "electron Landau regime." As β_p further increases, these contours become more convoluted, yielding a "finger" of increased damping at $k_\perp < k_\parallel < k_d$. Here damping is due primarily to the proton Landau resonance; we term this the "proton Landau regime."

Figure 2 represents the real parts of the proton cyclotron, electron Landau, and proton resonance factors at $\beta_p = 0.10$. If $|\zeta_j| > 3$ or $|\zeta_j^\pm| > 3$, the resonant $v_\parallel$ lies far from the thermal part of the velocity distribution $f(v_\parallel)$ (e. g., *Gary*, 1993); then the mode is nonresonant and the corresponding wave-particle interactions are weak. If the opposite sense of either inequality holds, it is a necessary but not sufficient condition for resonance and strong damping by the jth species. The approximately vertical contours of Figure 2(a) are similar in character to the damping contours of the $\beta_p = 0.10$ panel of Figure 1, suggesting that proton cyclotron damping is the primary mechanism here. We confirm this by noting that the condition $|\zeta_p^-| = 3.0$ lies at $k_\parallel c/\omega_p \simeq 0.6$ in Figure 2(a), which corresponds to the transition from weak to strong damping illustrated in the $\beta_p = 0.10$ panel of Figure 1.

Proton cyclotron damping begins abruptly as $k_\parallel$ increases, so it is appropriate to define the proton cyclotron dissipation wavenumber k_d as corresponding to this onset value of parallel wavenumber. To quantify this onset, we follow *Stawicki et al.* [2001] and fit the damping rate of Alfvén-cyclotron fluctuations at $\mathbf{k} \times \mathbf{B}_o = 0$ with the following trial function:

$$\frac{\gamma(k_\parallel)}{\Omega_p} = -m_1 \left(\frac{k_\parallel^2 c^2}{\omega_p^2} \right)^{m_2} \exp(-4 m_3^2 \omega_p^2 / k_\parallel^2 c^2) \tag{2}$$

where the m_j are fitting parameters. At $\mathbf{k} \times \mathbf{B}_o = 0$ and wavenumbers corresponding to -2 $\leq \gamma/\Omega_p \leq 0$ on the domain $10^{-4} \leq \beta_p \leq 1$ we obtain

$$m_1 = 0.66{\beta_p}^{0.43}$$

$$m_2 = 1.17 + 0.40{\beta_p}^{0.40}$$

$$m_3 = \frac{0.31}{{\beta_p}^{0.26}}$$

Proton cyclotron damping onsets at $k_\parallel \simeq m_3\omega_p/c$; we define this to be k_d. By fitting linear theory results to Equation (2) over a range of β_p, we obtain Figure 3. There is no simple power law scaling which covers all values of β_p illustrated here, but for limited ranges of this parameter we find

$$\frac{k_d c}{\omega_p} = \frac{0.44}{{\beta_p}^{0.21}} \qquad (10^{-4} \leq \beta_p \leq 10^{-2}) \tag{3a}$$

$$\frac{k_d c}{\omega_p} = \frac{0.26}{{\beta_p}^{0.40}} \qquad (10^{-1} \leq \beta_p \leq 2.0) \tag{3b}$$

Because $v_p/\Omega_p = \sqrt{\beta_p/2}c/\omega_p$, the scaling of the dissipation wavenumber relative to the thermal proton gyroradius follows immediately and is also plotted in Figure 3. The dissipation wavenumbers derived by *Gary* [1999] and *Gary and Nishimura* [2004] are based on the assumption of a fixed γ/Ω_p but have β_p scalings similar to that of Equation (3). Proton cyclotron damping at $\mathbf{k} \times \mathbf{B}_o = 0$ is independent of T_e/T_p, so the scaling of k_d should be independent of this temperature ratio.

Leamon et al. [1999, 2000] considered Alfvén-cyclotron fluctuations propagating at all angles relative to $\mathbf{B}_o$ and concluded that the dissipation wavenumber for kinetic Alfvén waves should scale as $k_d \sim \omega_p/c$. To determine whether the dissipation scale of magnetic turbulence is set by the ion inertial length scale or by the proton gyroradius, *Smith et al.* [2001] examined magnetic power spectra during an unusual low-β_e interval in the solar wind. Defining the dissipation wavenumber as corresponding to a distinct steepening or break in the magnetic power spectrum, they concluded that the ion inertial scale provided the better location of the dissipation wavenumber. Our results are consistent with these observations, predicting $k_d \sim \omega_p/c$ over $10^{-3} \lesssim \beta_p \lesssim 10^{-1}$; however, our prediction is derived from Alfvén-cyclotron waves with $k_\perp < k_\parallel$, rather than from the kinetic Alfvén waves considered by *Leamon et al.* [1999, 2000] which usually satisfy the opposite inequality.

At $k_\parallel \leq k_d$, proton cyclotron damping is exponentially weak, and other wave-particle interactions must be responsible for the damping of Alfvén-cyclotron fluctuations. Using $\omega_r \simeq k_\parallel v_A$ and the definition of ζ_j in Section 1,

$$\zeta_p \simeq \left(\frac{1}{\beta_p}\right)^{1/2} \qquad \text{and} \qquad \zeta_e \simeq \left(\frac{m_e}{m_p}\frac{1}{\beta_e}\right)^{1/2} \tag{4}$$

for the proton and electron Landau resonance factors. Figure 2 confirms these approximate scalings at $\beta_p = 0.10$. If we require that $|\zeta_j| < 3$ is a condition for Landau resonance, Equation (4) predicts that electrons can be Landau resonant with Alfvén-cyclotron fluctuations if $m_e/(10 m_p) < \beta_e$, and protons can Landau resonate if $0.10 \lesssim \beta_p$.

We now consider the regime $10^{-3} \lesssim \beta_e \lesssim 0.10$ and $k_\parallel < k_d$ where the electron Landau resonance is the dominant source of dissipation. Figure 4 plots the damping rate divided by the real frequency as a function of $k_\parallel$ and $k_\perp$ at $\beta_p = 0.10$; results at $\beta_p = 0.01$ and 0.001 are similar in that the contours of γ/ω_r are approximately independent of $k_\parallel$ at $k_\parallel < k_d$.

Figure 5 plots the damping decrement of obliquely propagating waves as a function of perpendicular wavenumber for a fixed parallel wavenumber $k_\parallel \simeq k_d/3$. For the parameters plotted here, we find that γ/ω_r scales as $k_\perp^2$. To quantify this relationship, we fit families of γ/ω_r curves to parabolas in $k_\perp c/\omega_p$; over $-0.01 \leq \gamma/\Omega_p \leq 0$ we obtain

$$\frac{\gamma(k_\perp)}{\omega_r} \simeq -0.35 \left(\frac{T_e\ m_e}{T_p\ m_p}\right)^{0.5} {\beta_p}^{0.5} \left(\frac{k_\perp c}{\omega_p}\right)^2 \tag{5}$$

This expression is valid at least for the following ranges of parameters: at $T_e/T_p = 1.0$ and $m_p/m_e = 1836$, $5 \times 10^{-4} < \beta_p < 0.1$; at $\beta_p = 0.01$ and $m_p/m_e = 1836$, $0.25 \leq T_e/T_p \leq 4.0$; and at $\beta_p = 0.01$ and $T_e/T_p = 1.0$, $100 \leq m_p/m_e \leq 1836$.

Although this equation has been derived for a prescribed value of $k_\parallel c/\omega_p$, Figure 5, other γ/ω_r plots for other values of β_e not shown here, and Fig. 4 of *Cranmer and van Ballegooijen* [2003] all indicate that this scaling is approximately valid for most wavenumbers such that $k_\parallel < k_d$. Earlier scalings for the damping rate of the kinetic Alfvén wave include $\gamma \sim \sqrt{\beta}$ by *Melrose* [1986], a $\beta_e^{1/2}$ dependence by *Tanaka et al.* [1989], and $\gamma \sim k_\perp^2$ by *Cranmer and van Ballegooijen* [2003]. Eq. (38) of *Gang* [1992] yields, after the correction of a typographical error, the same scalings with β_e, $k_\perp$ and m_p/m_e as our Equation (5).

At very long wavelengths solar wind fluctuation amplitudes are large, nonlinear turbulent interactions dominate, and the normal mode analysis used here may not be applicable. However, as k increases, the fluctuation amplitudes decrease, ω_r becomes greater than γ_c and normal modes become an appropriate means of describing the fluctuations. At still shorter wavelengths, $|\gamma|$ also becomes greater than γ_c and dissipation becomes becomes an important energy transfer process. In these latter two regimes Equations (2) and (5) can be used in models for turbulence in collisionless plasmas to determine how various damping mechanisms affect the fluctuating magnetic energy power spectra $W_B(\mathbf{k})$. Examples of such models include the diffusion model of *Zhou and Matthaeus* [1990] and the advection-diffusion equation of *Cranmer and van Ballegooijen* [2003]. Specifically, we suggest that Equation (5) with $\omega_r \simeq k_\parallel v_A$ would be an appropriate damping expression to use in solving the magnetic turbulence transport equation of *Cranmer and van Ballegooijen* [2003].

Figure 6 illustrates the contrast between the abrupt onset of proton cyclotron damping with $k_\parallel$ described by Equation (2) and the more gradual increase of dissipation with $k_\perp$ due to the electron Landau resonance described by Equation (5). For comparison, this figure also plots the energy cascade rate of Equation (1) rewritten as

$$\frac{\gamma_c}{\Omega_p} = a_c \left(kc/\omega_p\right)^{2/3} \tag{6}$$

with two different values of a_c. A crossing of the $\gamma(k)$ curve and the $\gamma_c(k)$ curve corresponds to a critical wavenumber at which the dominant energy transfer process changes from cascade to damping. The magnitude of the proton cyclotron damping rate $\gamma(k_\parallel)$ increases so abruptly with parallel wavenumber that this critical wavenumber is a very weak function of a_c; that is, it is approximately equal to k_d for a broad range of energy cascade rates. Thus, in this case, the wavenumber at onset of the dissipation range is determined by the local properties of the plasma. In contrast, the magnitude of the damping rate $\gamma(k_\perp)$ of the electron Landau resonance regime increases gradually, so that the critical wavenumber is a much more sensitive function of the cascade rate. This is analogous to the way in which the dissipation scale is determined by the Kolmogorov theory for fluid turbulence [e.g., *Frisch*, 1995]; the critical wavenumber is determined not only by the local parameters but also by the amplitude of the magnetic turbulence at large wavelengths.

We now consider the regime $0.10 \leq \beta_p \leq 10$ and $k_\parallel < k_d$ where the proton Landau resonance is the dominant damping mechanism at relatively small θ. In Figures 1d and 1e proton cyclotron damping still yields relatively vertical contours at $k_\parallel > k_d$. But the increased efficiency of the proton Landau resonance yields an increase of damping which appears as a finger pointing toward small $k_\parallel$ in the panels representing both $\beta_p = 1.0$ and 10. Furthermore, at $\beta_p = 10.0$ there is another finger corresponding to reduced damping pointing upward along the line $k_\perp = k_\parallel$. The contours of both fingers are convoluted functions of both $k_\parallel$ and $k_\perp$, so that we have not been able to obtain quantitative scaling relations in this case. What we have done is derive an approximate scaling relation as follows: for various values of wavenumber and β_p, we have found the maximum value of $|\gamma|$; for the parameters considered here the maximum damping rate is constrained by $0^o < \theta \lesssim 28^o$. From these maximum values we obtain

$$\frac{\gamma_{max}}{\Omega_p} \sim -0.10\beta_p \left(\frac{kc}{\omega_p}\right)^2$$

over $0.5 \lesssim \beta_p \lesssim 10$; at smaller values of β_p this damping decreases much faster than β_p.

For the wavenumber domain illustrated in Figure 1d, $0.5 \lesssim \mathrm{Re}(\zeta_p) \lesssim 1.0$; for Figure 1e, $0.1 \lesssim \mathrm{Re}(\zeta_p) \lesssim 0.3$. Thus the proton Landau resonance is clearly the source of the strong damping here. But the ζ_p are slowly changing functions of the wavevector in both cases, so the details of Figures 1d and 1e cannot be explained solely by invoking this resonance. To help understand these fingers, in the following section we examine the roles of the two distinct damping mechanisms associated with the Landau resonance.

3. Landau damping versus transit-time damping

The Landau resonance at $\omega_r = k_\parallel v_\parallel$ corresponds to two distinct types of wave-particle interactions [*Stix*, 1992]. If $\delta E_\parallel \neq 0$, the exchange of field and particle energy in a thermal plasma leads to Landau damping, the well-known mechanism for dissipation of electrostatic wave energy. However, if $\delta B_\parallel \neq 0$, the interaction of the magnetic moment of a charged particle with the parallel gradient of the magnetic field leads to "transit-time magnetic damping" [*Barnes*, 1966]. The distinct character of these two interactions suggests that they yield distinct plasma signatures. Although simulations have examined the electron Landau resonance of Alfvén-cyclotron fluctuations at relatively low β_e [*Tanaka et al.*, 1987, 1989; *Geary et al.*, 1990; *Gary and Nishimura*, 2004], we are not aware of any simulations which have considered the ion Landau resonances at higher β values so that it is appropriate to lay the groundwork for future such studies.

To discern the parameter domains over which proton Landau damping and proton transit-time damping are likely to dominate collisionless dissipation, we constructed contour plots for the parallel fluctuating electric field ratio $|\delta E_\parallel|^2/|\delta \mathbf{E}|^2$ and the parallel fluctuating magnetic field ratio $|\delta B_\parallel|^2/|\delta \mathbf{B}|^2$. Figure 7 is a plot of these two ratios at $\beta_p = 0.10$; Figure 8 illustrates the same two ratios at $\beta_p = 10$. Although $|\delta E_\parallel|^2 = 0$ at $\mathbf{k} \times \mathbf{B}_o = 0$, Figure 7 shows that $|\delta E_\parallel|^2/|\delta \mathbf{E}|^2$ is a monotonically increasing function of $k_\parallel$ (at fixed $k_\perp$) and $k_\perp$ (at fixed $k_\parallel$). Similarly, the parallel fluctuating electric field ratio of Figure 8 is almost a monotonic function of both $k_\parallel$ and $k_\perp$. So Landau damping on either species is not the likely mechanism for the nonmonotonic contours of damping at $k_\perp c/\omega_p \leq 0.10$ in Figures 1d and 1e. In contrast the parallel fluctuating magnetic field ratios of Figure 7 and Figure 8 are nonmonotonic as a function of either $k_\parallel$ or $k_\perp$, and the fingers of Figure 8b bear a strong resemblance to the fingers of Figure 1e, suggesting that proton transit-time damping is important at high β_p.

Here we derive an expression to evaluate the relative efficacy of transit-time magnetic damping versus Landau damping for a plasma species j. Following *Stix* [1992], we compare the force due to the interaction of the magnetic moment of a charged particle with the parallel gradient of the background magnetic field,

$$m_j \frac{dv_\parallel}{dt} = -\frac{m_j v_\perp^2}{2B_o} k_\parallel |\delta B_\parallel|$$

with the force due to the fluctuating parallel electric field

$$m_j \frac{dv_\parallel}{dt} = e_j \delta E_\parallel$$

We define the jth species efficacy ratio R_j as the square of the ratio of these two forces; then assuming isotropy and averaging over the velocity distribution we obtain

$$R_j(\mathbf{k}) = \frac{1}{4}\frac{v_j^2}{c^2}\left(\frac{k_\parallel v_j}{\Omega_j}\right)^2 \frac{|\delta B_\parallel(\mathbf{k})|^2}{|\delta E_\parallel(\mathbf{k})|^2} \tag{7}$$

We further define the part of $R_j(\mathbf{k})$ which contains the fluctuation properties as

$$\rho(\mathbf{k}) \equiv \left(\frac{k_\parallel c}{\omega_p}\right)^2 \frac{|\delta B_\parallel|^2}{|\delta E_\parallel|^2} \tag{8}$$

Figures 7b and 8b indicate that contours of constant $|\delta B_\parallel|^2/|\delta \mathbf{B}|^2$ run approximately parallel to lines of constant θ, so we here consider the direction of propagation as the primary independent variable. We define quasi-parallel (q-$\|$) propagation as the limit of $\theta \to 0$, and quasi-perpendicular (q-$\perp$) propagation as corresponding to $\theta \simeq 85^o$.

We first evaluated Equation (7) for the electron Landau regime. Figure 9a shows that for most angles of propagation, $\rho(\mathbf{k})$ at $\beta_e = 0.01$ is maximum in the twin limits of $\theta \to 0$ and $kc/\omega_p \to 0$. Choosing values at $kc/\omega_p = 0.10$ and in the zero θ limit, we plot the resulting quasi-parallel maximum value of ρ versus β_e in Figure 9b; the results are well fit by

$$\rho_{max}(q - \|) = 3.14 \left(\frac{c}{v_A}\right)^2 \frac{1}{{\beta_e}^2} \qquad (10^{-3} \lesssim \beta_e \lesssim 1) \tag{9}$$

where we have obtained the $(c/v_A)^2$ scaling from a separate series of computations. Thus the approximate scaling of the maximum value of the electron efficacy ratio is

$$R_e \simeq \left(\frac{\pi}{16}\right) \qquad (10^{-3} \lesssim \beta_p \lesssim 1)$$

Therefore, electron transit-time damping may compete with electron Landau damping as a dissipation mechanism for Alfvén-cyclotron fluctuations primarily at quasi-parallel propagation. Under the kinetic Alfvén wave conditions of relatively oblique propagation, Figure 9a indicates that electron Landau damping is the dominant damping mechanism.

Next we evaluated Equation (7) in the proton Landau regime, that is, at $k_\parallel < k_d$ and $0.10 \lesssim \beta_p \leq 10$. Figure 10 shows that in general there is a second relative maximum for $\rho(\theta)$ at strongly oblique propagation, and at $\beta_p \gtrsim 1$, this quasi-perpendicular maximum can be larger than the $\rho_{max}(q-\|)$ at smaller values of β_p. Our computations show that the $\rho(q-\perp)$ scales, like $\rho(q-\|)$, as $(c/v_A)^2$, but unlike Equation (9) this quantity is a relatively weak function of β_p. Then the primary β_p dependence of $R_p(q-\perp)$ is approximately the β_p^2 dependence implied by the v_j^4 factor of Equation (7). We get $R_p(q-\perp) > 1$ over $2 < \beta_p \leq 10$; for this range of β_p proton transit-time damping is stronger than proton Landau damping both at quasi-perpendicular and (via Figure 10) quasi-parallel propagation of Alfvén-cyclotron fluctuations.

Table 1. Alfvén-cyclotron damping regimes

Resonance Regime	Wavenumber Range	β Range
Proton Cyclotron	$k_d < k_\parallel$	$10^{-3} \leq \beta_p \leq 10$
Electron Landau	$k_\parallel < k_d$	$10^{-3} \leq \beta_e \lesssim 0.10$
Proton Landau	$k_\parallel < k_d$	$0.10 \lesssim \beta_p \leq 10$

4. Conclusions

We have used full Vlasov linear dispersion theory in a homogeneous, magnetized plasma to study the damping of Alfvén-cyclotron fluctuations in an electron-proton plasma over a broad range of β_p values. By examining the $k_\perp$ versus $k_\parallel$ contour plots of the damping and cyclotron resonance factors of these fluctuations, we have demonstrated that there are three distinct regimes for the damping of these waves; these are summarized in Table 1. We have derived analytic expressions for $k_d c/\omega_p$ describing the abrupt onset of proton cyclotron damping [Equation (2)], and for the damping rate due to the electron Landau resonance [Equation (5)]. We have furthermore studied the competition between transit-time damping and Landau damping in the two Landau resonance regimes; we find that magnetic transit-time damping is relatively important for electrons only at quasi-parallel propagation, but that at sufficiently high β_p proton transit-time damping can be important at both quasi-parallel and quasi-perpendicular propagation.

We have interpreted our results in terms of their possible observational consequences for magnetic power spectra of cascading Alfvén-cyclotron fluctuations. Proton cyclotron damping of Alfvén-cyclotron fluctuations has an abrupt onset with increasing parallel wavenumber; the resulting critical wavenumber is almost independent of the cascade rate. Thus, if this mechanism is the primary source of fluctuation damping, the transition from the inertial range to the dissipation range should be primarily a function of local plasma parameters, specifically β_p, and should scale as $k_d \sim \omega_p/c$ over $10^{-3} \lesssim \beta_p \lesssim 10^{-1}$. The electron Landau resonance is the most important damping mechanism for Alfvén-cyclotron fluctuations at $k_\parallel < k_d$ in plasmas with $\beta_p \lesssim 0.10$. The damping of these kinetic Alfvén waves increases gradually, as in Equation (5), implying that, if the electron Landau resonance is the primary source of fluctuation dissipation, the transition from inertial range to dissipation range should be a sensitive function of the turbulent energy cascade rate, and thereby a function of the turbulence amplitude at large scales, which is a nonlocal quantity.

Although both *Leamon et al.* [2000] and we associate the onset of the dissipation range with a wavenumber which scales as the ion inertial length, we disagree as to how this conclusion is reached. *Leamon et al.* [1998a] examined solar wind magnetic fluctuation spectra in both the inertial and dissipation ranges, and concluded that "parallel-propagating waves, such as Alfvén waves, are inconsistent with the data." *Leamon et al.* [2000] further stated: "Solar wind observational evidence suggest the relevance of the ion inertial scale..." and concluded that "...a significant fraction of dissipation in the corona and solar wind likely proceeds through a perpendicular cascade and small-scale reconnection, coupled to kinetic processes that act at oblique wavevectors." We have theoretically derived a dissipation wavenumber which scales with the ion inertial length from fluctuations which propagate at $\mathbf{k} \times \mathbf{B}_o = 0$; furthermore, we associate the onset of dissipation of obliquely propagating Alfvén-cyclotron waves with the fundamentally nonlocal amplitude of the turbulent spectrum. These differences suggest that the question of scaling for the onset of dissipation of Alfvén-cyclotron turbulence is worthy of further study, via both observations and computer simulations.

Figure 11 is a cartoon illustrating our current view concerning the onset of the dissipation range. The left-hand panel indicates the consequences of proton cyclotron damping where k_d is independent of the cascade rate and the turbulence amplitude. The right-hand panel suggests that the wavenumber at which electron Landau damping dominates the cascade rate is a function of the turbulence amplitude; stronger turbulence pushes the onset of the dissipation range to shorter wavelengths. The right-hand panel of Figure 11 is analogous to the case of turbulence in Navier-Stokes fluids, whereas the left-hand panel represents a result which is specific (and may be unique) to collisionless plasmas [e.g., *Li et al.*, 2001].

Our results are based upon the assumption that the plasma consists only of electrons and protons. Of course, the solar corona, the solar wind, and most space and astrophysical plasmas bear minor ions which are usually relatively tenuous compared to the protons but which nevertheless may have important consequences for wave-particle scattering at the low frequencies considered here. A natural extension of this research would be to include heavy ions such as alpha particles as a minority species. Another obvious extension of this work would be to study the damping of the magnetosonic-whistler mode and, if $T_e/T_p > 1$, the ion acoustic mode.

Appendix

The fluctuation energy cascade rate stated as Equation (1) can be derived from Kolmogorov-type arguments for turbulent fluids. For isotropic turbulence in three dimensions, two fundamental assumptions are made. First, it is assumed that the energy flux through k space is a constant

$$\frac{v_k^2}{\tau_k} = C_1 \quad , \qquad (A-1)$$

where $k = |\mathbf{k}|$ is the omnidirectional wavenumber, v_k is the RMS fluctuation velocity in a wavenumber band near wavenumber k, τ_k is the energy-transfer timescale at wavenumber k, and C_1 is a constant. Second, it is assumed that the energy-transfer time τ_k is proportional to the eddy-turnover time (eddy lifetime) L/v, which is $1/kv_k$ for an eddy of scale $L = k^{-1}$. This second assumption is written

$$\tau_k \quad = \quad C_2 \, \frac{1}{v_k k} \qquad (A-2)$$

where C_2 is a numerical constant of order unity. Combining expressions (A-1) and (A-2) to eliminate τ_k yields

$$v_k \quad = \quad (C_1 C_2)^{1/3} \; k^{-1/3} \qquad . \qquad (A-3)$$

Utilizing equation (A-3), equation (A-2) becomes

$$\tau_k \quad = \quad C_2 \; (C_1 C_2)^{-1/3} \; k^{-2/3} \qquad . \qquad (A-4)$$

Expression (A-3) is used to evaluate $(C_1 C_2)^{1/3}$ at the integral scale ($k = k_o$), yielding

$$(C_1 C_2)^{1/3} \quad = \quad v_o k_o^{1/3} \qquad (A-5)$$

where v_o is the amplitude of the turbulent fluctuations at the integral scale (at large scales) and where k_o is the wavenumber of the integral scale ($L_o = k_o^{-1}$ is the correlation length of the turbulence). Using equation (A-5), equation (A-4) for the energy-transfer timescale τ_k becomes

$$\tau_k \quad = \quad C_2 \; v_o^{-1} \; k_o^{-1/3} \; k^{-2/3} \qquad . \qquad (A-6)$$

In expression (A-6) C_2 is a constant of order unity, and v_o and k_o are properties of the amplitude of the turbulence at large scales [cf. eq. (7.8) of *Frisch*, 1995; eq. (6.11) of *Pope*, 2000]. Note that if the amplitude of the turbulence v_o increases, then the energy-transfer timescale decreases.

Acknowledgments. We acknowledge useful conversations with Steve Cranmer, Aad van Ballegooijen, Phil Isenberg, and Steve Spangler. This work was performed at Los Alamos National Laboratory under the auspices of the U.S. Department of Energy (DOE) and was supported by the DOE Office of Basic Energy Sciences, Division of Engineering and Geosciences, and the Sun-Earth Connection Guest Investigator Program of the National Aeronautics and Space Administration.

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Figure Captions

Figure 1. The damping rate (γ/Ω_p) of Alfvén-cyclotron fluctuations as a function of the parallel and perpendicular components of the wavevector at five values of β_p as labeled.

Figure 2. (a) The real part of the proton cyclotron resonance factor ζ_p^-, (b) the real part of the electron Landau resonance factor ζ_e, and (c) the real part of the proton Landau resonance factor ζ_p for Alfvén-cyclotron fluctuations as functions of the parallel

and perpendicular components of the wavevector at $\beta_p = 0.10$.

Figure 3. The proton cyclotron dissipation wavenumber for Alfvén-cyclotron fluctuations at $\mathbf{k} \times \mathbf{B}_o = 0$ as a function of β_p. The solid line represents $k_d c/\omega_p$ and the dashed line represents $k_d v_p/\Omega_p$.

Figure 4. The damping rate divided by the real frequency (γ/ω_r) of Alfvén-cyclotron fluctuations as a function of the parallel and perpendicular components of the wavevector at $\beta_p = 0.10$.

Figure 5. The damping rate divided by the real frequency (γ/ω_r) of Alfvén-cyclotron fluctuations as a function of perpendicular wavenumber for three values of β_e as labeled. Here $k_\| \simeq k_d/3$ and $-0.01 \leq \gamma/\Omega_p \leq 0$.

Figure 6. The line with solid dots represents $\gamma(k_\|)$, the damping rate of Alfvén-cyclotron fluctuations at $\mathbf{k} \times \mathbf{B}_o = 0$ in a $\beta_p = 0.01$ plasma. The line with open dots represents $\gamma(k_\perp)$, the damping rate of Alfvén-cyclotron fluctuations at $k_\| c/\omega_p = 0.8$ in a $\beta_p = 0.01$ plasma. The two dashed lines represent the assumed cascade rate of Equation (1) with different values of a_c as labeled.

Figure 7. (a) The parallel fluctuating electric field ratio and (b) the parallel fluctuating magnetic field ratio of Alfvén-cyclotron fluctuations as functions of the parallel and perpendicular components of the wavevector at $\beta_p = 0.10$.

Figure 8. (a) The parallel fluctuating electric field ratio and (b) the parallel fluctuating magnetic field ratio of Alfvén-cyclotron fluctuations as functions of the parallel and perpendicular components of the wavevector at $\beta_p = 10$.

Figure 9. (a) The quantity $\rho(\mathbf{k})$ defined by Equation (8) as a function of θ for three different wavenumbers of Alfvén-cyclotron fluctuations at $\beta_p = 0.01$. (b) The dots represent maximum values of $\rho(\mathbf{k})$ (that is, the value in the limits of vanishing θ and vanishing wavenumber) as a function of β_p. The dashed line represents Equation (9).

Figure 10. The quantity $\rho(\mathbf{k})$ defined by Equation (8) as a function of θ for three different values of β_p for Alfvén-cyclotron fluctuations at $kc/\omega_p = 0.05$.

Figure 11. Cartoon illustrating possible differences between turbulent magnetic power spectra which are subject to two different types of damping. The left hand panel illustrates the possible consequences of the abrupt onset of proton cyclotron damping. The right hand panel shows the possible consequences of the more gradual onset of electron Landau damping.